Unit 11 Day 1

NAME_____________________Period___

Factorials, Counting Principle

Factorials, Counting Principle

& Simple Probability

I can______________________________

\[
\begin{array}{l}
0! = 1 \\
1! = 1 \\
2! = 2 \\
3! = 6 \\
4! = 24 \\
5! = 120 \\
6! = 720 \\
\end{array}
\]
How many ways can 5 paintings be lined up on a wall?

\[5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120\]
Factorials!

Factorials are simple things. They're just products, indicated by an exclamation point. For instance, "three factorial" is written as 3! and means \( 1 \cdot 2 \cdot 3 = 6 \)

The factorial function (symbol: \( ! \)) means to multiply a series of descending natural numbers. Examples:

- \( 4! = 4 \times 3 \times 2 \times 1 = 24 \)
- \( 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040 \)
- \( 1! = 1 \)

Let's practice. (Find the factorial key on your calculator.)

\[
\begin{align*}
6! &= 720 \\
12! &= 479,001,600 \\
6! / 4! &= \\
17! / (14! 3!) &=
\end{align*}
\]
When would we need to use a factorial?

Use a factorial to count how many possible ways you could organize your books. In this case, you could organize your books in "six factorial" different arrangements.

The mathematical sign for factorial is "!" (that doesn't mean to shout the number excitedly)

"Six factorial" = 6!

Let's try some:

In how many ways can 8 bicycles be parked in a row?

\[ 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40,320 \]

How many ways can you rearrange the letters in the work HELP?

\[ 4 \cdot 3 \cdot 2 \cdot 1 = 24 \]

How many ways can I display my 6 trophy's on the shelf if they all different?

\[ 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720 \]
Counting Principle

Example: You are buying a new car.
There are 2 body styles:

- sedan or hatchback

There are 5 colors available:

- Black
- Red
- Green
- Blue
- Light blue

There are 3 models:

- GL (standard model),
- SS (sports model with bigger engine),
- SL (luxury model with leather seats)

How many total choices?

\[ \text{Total Choices} = 2 \times 5 \times 3 = 30 \]

Jenny has nine different skirts, seven different tops, ten different pairs of shoes, two necklaces and five bracelets. In how many ways can Jenny dress up?

\[ 9 \cdot 7 \cdot 10 \cdot 2 \cdot 5 = \]
Counting Principle Practice:

1. If 12 people are running for office, how many ways can we choose a President, Vice President and a Secretary? \( \binom{12}{1} \cdot \binom{11}{1} \cdot \binom{10}{1} = 1,320 \)

2. Derek must choose a four-digit PIN number. Each digit can be chosen form 0-9. How many different possible PIN numbers can Derek choose? \( 10 \cdot 10 \cdot 10 \cdot 10 = 10,000 \)

3. S= School  
   T=Town  
   H=Home

   How many ways can Ben get home if he passes through the town center?

   \( 3 \cdot 5 = 15 \)

4. Your state issues license plates consisting of letters and numbers. There are 26 letters and the letters may be repeated. There are 10 digits and the digits may be repeated. How many possible license plates can be issued with two letters followed by three numbers?

   \( 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \)
Simple Probability

Tossing a Coin

When a coin is tossed, there are two possible outcomes:

- heads (H) or
- tails (T)

We say that the probability of the coin landing H is \( \frac{1}{2} \).

And the probability of the coin landing T is \( \frac{1}{2} \).

Throwing Dice

When a single die is thrown, there are six possible outcomes: 1, 2, 3, 4, 5, 6.

The probability of any one of them is \( \frac{1}{6} \).

\[ P(4) = \frac{1}{6} \]

Probability Line

You can show probability on a Probability Line:

- Impossible
- Unlikely
- Even Chance
- Likely
- Certain

Probability is always between 0 and 1

1-in-6 Chance

4-in-5 Chance
Vocabulary:

**Theoretical Probability** the ratio of the number of favorable outcomes to the number of possible outcomes if all outcomes have the same chance of happening.

(the mathematical calculation of an event occurring)

**Experimental Probability** is based on repeated trials of an experiment or # of Successes / # of Trials.

(actually running the experiment, like rolling the dice)

**Sample Space** is a collection of all possible outcomes of a random experiment.

(for a die it is 1,2,3,4,5,6)

**Questions:**

Is it possible to flip a coin and get 6 heads in a row? ____________ Is that likely to happen? ______

What is the sample space for this spinner? __________________________________________
A fair die is rolled.

What is the sample space __________________________

p(6)=_______ p(7)=_______

Find the probability of showing multiple of 3 ______________________

Find the probability of showing divisors of 8 ________________

p(not a 2) _________ p (number less than 5)=_____

Bag of Marbles: 5 red, 6 blue, 2 white, 2 green

What color is most likely to be picked? ______

What is the sample space? __________________________

What is the probability of choosing a white? p(white)?_____

p(green)?___________ p(red)?__________

p(pink)?__________ P(blue)?________
\[ P(A) = \frac{2}{5} \]
\[ P(A') = \frac{3}{5} \]

A' \times \overline{A}
Complementary Events

You roll a die. what is the probability that a 2 will appear? \( \frac{1}{6} \)

What is the probability that a 2 will not appear? \( \frac{5}{6} \)

These are considered Complementary Events. If you add up the results for both of them they will always equal 1.

Bag of Marbles: 3 red, 4 blue, 2 orange

Find the probability for the event and its complement.

\[
p(\text{red}) = \frac{3}{9} = \frac{1}{3} \\
p(\text{not orange}) = \frac{2}{3} \\
p(\text{white})
\]
It was found that 45 students own a laptop and 27 of these students do not own a four-wheeler. There were 35 students that do not own a laptop and 12 of these students own a four-wheeler.

<table>
<thead>
<tr>
<th>Fill in the chart</th>
<th>Students with Laptops</th>
<th>Students without Laptops</th>
<th>Total:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students who own Four-Wheelers</td>
<td>18</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>Students who do not own Four-Wheelers</td>
<td>27</td>
<td>23</td>
<td>50</td>
</tr>
<tr>
<td>Total:</td>
<td>45</td>
<td>35</td>
<td>80</td>
</tr>
</tbody>
</table>

\[
p(\text{students with laptop}) = \frac{45}{80} = \frac{9}{16}
\]

\[
p(\text{students who do not own 4 wheelers}) = \frac{20}{80} = \frac{5}{8}
\]

\[
p(\text{students with both laptop and four-wheeler}) = \frac{18}{80} = \frac{9}{40}
\]
How many ways can you arrange the letters in the word **BLUE**?

How many 2 digit numbers can you make using the digits 1, 2, 3 and 4 without repeating the digits?

\[ 4 \cdot 3 = 12 \]
A shelf can hold 7 trophies. How many ways can the trophies be arranged if there are 10 trophies available?

\[ 7 \cdot 6 \cdot 5 = 210 \]

If 7 athletes are running the 100m dash, how many ways can we have a first place, second place and third place?

\[ 10 \ \text{Pr} \ 7 \]

What is the probability of picking an Ace from a standard deck of 52 cards? __________

What is the probability of picking a red card? _____

What is the probability of picking a spade? _____
7, 4, 9, 2, 5, 8